

Найти сумму ряда

$$\sum_{n=2}^{\infty} \left(\frac{1}{n^2+n-2} \right)$$

$$\frac{1}{n^2+n-2} = \frac{1}{(n+2)(n-1)} = \frac{A}{n+2} + \frac{B}{n-1} = \frac{A(n-1)+B(n+2)}{(n+2)(n-1)}$$

$$An - A + Bn + 2B \Rightarrow \begin{cases} 2B - A = 1 \\ A + B = 0 \end{cases} \Rightarrow \begin{cases} 3B = 1 \\ A = -B \end{cases} \Rightarrow \begin{cases} B = \frac{1}{3} \\ A = -\frac{1}{3} \end{cases}$$

$$\Rightarrow \frac{1}{n^2+n-2} = \frac{1}{3(n-1)} - \frac{1}{3(n+2)} = \frac{1}{3} \left(\frac{1}{n-1} - \frac{1}{n+2} \right)$$

$$\Rightarrow \sum_{n=2}^{\infty} \frac{1}{n^2+n-2} = \frac{1}{3} \left(\sum_{n=2}^{\infty} \frac{1}{n-1} - \sum_{n=2}^{\infty} \frac{1}{n+2} \right) =$$

$$\frac{1}{3} \sum_{n=2}^{\infty} \left(\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots \right) - \frac{1}{4} - \frac{1}{5} - \frac{1}{6} - \frac{1}{7} - \dots \right) = \frac{1}{3} \left(1 + \frac{1}{2} + \frac{1}{3} \right) =$$
$$= \frac{1}{3} \cdot \frac{11}{6} = \frac{11}{18}$$