Даны координаты пирамиды: A1(2,3,1), A2(4,1,-2), A3(6,3,7), A4(-5,-4,8)  
**1) Координаты векторов**.  
Координаты векторов находим по формуле:  
X = xj - xi; Y = yj - yi; Z = zj - zi  
здесь X,Y,Z координаты вектора; xi, yi, zi - координаты точки Аi; xj, yj, zj - координаты точки Аj;  
Например, для вектора A1A2  
X = x2 - x1; Y = y2 - y1; Z = z2 - z1  
X = 4-2; Y = 1-3; Z = -2-1  
A1A2(2;-2;-3)  
A1A3(4;0;6)  
A1A4(-7;-7;7)  
A2A3(2;2;9)  
A2A4(-9;-5;10)  
A3A4(-11;-7;1)  
**2) Модули векторов** (длина ребер пирамиды)  
Длина вектора a(X;Y;Z) выражается через его координаты формулой:  
https://chart.googleapis.com/chart?cht=tx&chl=|a|%20=%20\sqrt%7bX%5e%7b2%7d%20%2B%20Y%5e%7b2%7d%20%2B%20Z%5e%7b2%7d%7d  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b1%7dA_%7b2%7d|%20=%20\sqrt%7b2%5e%7b2%7d%20%2B%202%5e%7b2%7d%20%2B%203%5e%7b2%7d%7d%20=%20\sqrt%7b17%7d%20=%204.123  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b1%7dA_%7b3%7d|%20=%20\sqrt%7b4%5e%7b2%7d%20%2B%200%5e%7b2%7d%20%2B%206%5e%7b2%7d%7d%20=%20\sqrt%7b52%7d%20=%207.211  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b1%7dA_%7b4%7d|%20=%20\sqrt%7b7%5e%7b2%7d%20%2B%207%5e%7b2%7d%20%2B%207%5e%7b2%7d%7d%20=%20\sqrt%7b147%7d%20=%2012.124  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b2%7dA_%7b3%7d|%20=%20\sqrt%7b2%5e%7b2%7d%20%2B%202%5e%7b2%7d%20%2B%209%5e%7b2%7d%7d%20=%20\sqrt%7b89%7d%20=%209.434  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b2%7dA_%7b4%7d|%20=%20\sqrt%7b9%5e%7b2%7d%20%2B%205%5e%7b2%7d%20%2B%2010%5e%7b2%7d%7d%20=%20\sqrt%7b206%7d%20=%2014.353  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b3%7dA_%7b4%7d|%20=%20\sqrt%7b11%5e%7b2%7d%20%2B%207%5e%7b2%7d%20%2B%201%5e%7b2%7d%7d%20=%20\sqrt%7b171%7d%20=%2013.077  
**4) Площадь грани**  
Площадь грани можно найти по формуле:  
https://chart.googleapis.com/chart?cht=tx&chl=S%20=%20\frac%7b1%7d%7b2%7d%20|a|\cdot%20|b|%20sin%20\gamma  
где  
https://chart.googleapis.com/chart?cht=tx&chl=sin%20\gamma%20%20=%20\sqrt%7b1%20-%20cos%20\gamma%5e%7b2%7d%7d  
Найдем площадь грани A1A2A3  
Найдем угол между ребрами A1A2(2;-2;-3) и A1A3(4;0;6):  
https://chart.googleapis.com/chart?cht=tx&chl=cos%20\gamma%20%20%20=%20\frac%7b2\cdot%204%20%2B%20(-2)\cdot%200%20%2B%20(-3)\cdot%206%7d%7b\sqrt%7b17%7d\cdot%20\sqrt%7b52%7d%7d%20=%20-0.336  
https://chart.googleapis.com/chart?cht=tx&chl=sin%20\gamma%20%20=%20\sqrt%7b1%20-%200.336%5e%7b2%7d%7d%20=%200.942  
Площадь грани A1A2A3  
  
Найдем площадь грани с учётом геометрического смысла векторного произведения:  
https://chart.googleapis.com/chart?cht=tx&chl=S%20=%20\frac%7b1%7d%7b2%7d%20|\overline%7bA_%7b1%7dA_%7b2%7d%7d%20\cdot%20%20\overline%7bA_%7b1%7dA_%7b3%7d%7d|  
Векторное произведение:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | i | j | k | | 2 | -2 | -3 | | 4 | 0 | 6 | |  | | = |

=i((-2)\*6-0(-3)) - j(2\*6-4(-3)) + k(2\*0-4(-2)) = -12i - 24j + 8k  
  
**5) Объем пирамиды**.  
Объем пирамиды, построенный на векторах a1(X1;Y1;Z1), a2(X2;Y2;Z2), a3(X3;Y3;Z3) равен:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=V%20=%20\frac%7b1%7d%7b6%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | X1 | Y1 | Z1 | | X2 | Y2 | Z2 | | X3 | Y3 | Z3 | |  | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=V%20=%20\frac%7b1%7d%7b6%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | 2 | -2 | -3 | | 4 | 0 | 6 | | -7 | -7 | 7 | |  | | https://chart.googleapis.com/chart?cht=tx&chl=%20=%20\frac%7b308%7d%7b6%7d%20=%2051.333 |

Находим определитель матрицы  
∆ = 2\*(0\*7-(-7)\*6)-4\*((-2)\*7-(-7)\*(-3))+(-7)\*((-2)\*6-0\*(-3)) = 308  
**7) Уравнение прямой**  
Прямая, проходящая через точки A1(x1; y1; z1) и A2(x2; y2; z2), представляется уравнениями:  
https://chart.googleapis.com/chart?cht=tx&chl=\frac%7bx%20-%20x_%7b1%7d%7d%7bx_%7b2%7d%20-%20x_%7b1%7d%7d%20=%20\frac%7by%20-%20y_%7b1%7d%7d%7by_%7b2%7d%20-%20y_%7b1%7d%7d%20=%20\frac%7bz%20-%20z_%7b1%7d%7d%7bz_%7b2%7d%20-%20z_%7b1%7d%7d  
Параметрическое уравнение прямой:  
x=x0+lt  
y=y0+mt  
z=z0+nt  
Уравнение прямой A1A2(2,-2,-3)  
https://chart.googleapis.com/chart?cht=tx&chl=\frac%7bx%20-%202%7d%7b2%7d%20=%20\frac%7by%20-%203%7d%7b-2%7d%20=%20\frac%7bz%20-%201%7d%7b-3%7d  
Параметрическое уравнение прямой:  
x=2+2t  
y=3-2t  
z=1-3t  
**8) Уравнение плоскости**.  
Если точки A1(x1; y1; z1), A2(x2; y2; z2), A3(x3; y3; z3) не лежат на одной прямой, то проходящая через них плоскость представляется уравнением:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | x-x1 | y-y1 | z-z1 | | x2-x1 | y2-y1 | z2-z1 | | x3-x1 | y3-y1 | z3-z1 | |  | | = 0 |

Уравнение плоскости A1A2A3

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | x-2 | y-3 | z-1 | | 2 | -2 | -3 | | 4 | 0 | 6 | |  | | = 0 |

(x-2)((-2)\*6-0(-3)) - (y-3)(2\*6-4(-3)) + (z-1)(2\*0-4(-2)) = -12x - 24y + 8z + 88 = 0  
Упростим выражение: -3x - 6y + 2z + 22 = 0