Даны координаты пирамиды: A1(2,3,1), A2(4,1,-2), A3(6,3,7), A4(-5,-4,8)
**1) Координаты векторов**.
Координаты векторов находим по формуле:
X = xj - xi; Y = yj - yi; Z = zj - zi
здесь X,Y,Z координаты вектора; xi, yi, zi - координаты точки Аi; xj, yj, zj - координаты точки Аj;
Например, для вектора A1A2
X = x2 - x1; Y = y2 - y1; Z = z2 - z1
X = 4-2; Y = 1-3; Z = -2-1
A1A2(2;-2;-3)
A1A3(4;0;6)
A1A4(-7;-7;7)
A2A3(2;2;9)
A2A4(-9;-5;10)
A3A4(-11;-7;1)
**2) Модули векторов** (длина ребер пирамиды)
Длина вектора a(X;Y;Z) выражается через его координаты формулой:







**4) Площадь грани**
Площадь грани можно найти по формуле:

где

Найдем площадь грани A1A2A3
Найдем угол между ребрами A1A2(2;-2;-3) и A1A3(4;0;6):


Площадь грани A1A2A3

Найдем площадь грани с учётом геометрического смысла векторного произведения:

Векторное произведение:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
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|

|  |  |  |
| --- | --- | --- |
| i | j | k |
| 2 | -2 | -3 |
| 4 | 0 | 6 |

 |  |

 | = |

=i((-2)\*6-0(-3)) - j(2\*6-4(-3)) + k(2\*0-4(-2)) = -12i - 24j + 8k

**5) Объем пирамиды**.
Объем пирамиды, построенный на векторах a1(X1;Y1;Z1), a2(X2;Y2;Z2), a3(X3;Y3;Z3) равен:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=V%20=%20\frac%7b1%7d%7b6%7d |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| X1 | Y1 | Z1 |
| X2 | Y2 | Z2 |
| X3 | Y3 | Z3 |

 |  |

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| https://chart.googleapis.com/chart?cht=tx&chl=V%20=%20\frac%7b1%7d%7b6%7d |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| 2 | -2 | -3 |
| 4 | 0 | 6 |
| -7 | -7 | 7 |

 |  |

 | https://chart.googleapis.com/chart?cht=tx&chl=%20=%20\frac%7b308%7d%7b6%7d%20=%2051.333 |

Находим определитель матрицы
∆ = 2\*(0\*7-(-7)\*6)-4\*((-2)\*7-(-7)\*(-3))+(-7)\*((-2)\*6-0\*(-3)) = 308
**7) Уравнение прямой**
Прямая, проходящая через точки A1(x1; y1; z1) и A2(x2; y2; z2), представляется уравнениями:

Параметрическое уравнение прямой:
x=x0+lt
y=y0+mt
z=z0+nt
Уравнение прямой A1A2(2,-2,-3)

Параметрическое уравнение прямой:
x=2+2t
y=3-2t
z=1-3t
**8) Уравнение плоскости**.
Если точки A1(x1; y1; z1), A2(x2; y2; z2), A3(x3; y3; z3) не лежат на одной прямой, то проходящая через них плоскость представляется уравнением:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| x-x1 | y-y1 | z-z1 |
| x2-x1 | y2-y1 | z2-z1 |
| x3-x1 | y3-y1 | z3-z1 |

 |  |

 | = 0 |

Уравнение плоскости A1A2A3

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| x-2 | y-3 | z-1 |
| 2 | -2 | -3 |
| 4 | 0 | 6 |

 |  |

 | = 0 |

(x-2)((-2)\*6-0(-3)) - (y-3)(2\*6-4(-3)) + (z-1)(2\*0-4(-2)) = -12x - 24y + 8z + 88 = 0
Упростим выражение: -3x - 6y + 2z + 22 = 0