Решите уравнение: cosx+sin3x=sin4x+cos4x+1.

$$\cos x + \sin 3x = \sin 4x + \cos 4x + 1 \Leftrightarrow \sin 3x + \cos x = 2\sin 2x \cos 2x + 2\cos^2 x \Leftrightarrow$$
$$\Leftrightarrow \sin 3x + \cos x - 2\cos 2x(\sin 2x + \cos 2x) = 0$$

Дальше потребуется формула $\sin\alpha + \cos\beta = 2\sin\left(\frac{\alpha-\beta}{2} + \frac{\pi}{4}\right)\cdot\cos\left(\frac{\alpha+\beta}{2} - \frac{\pi}{4}\right)$.

$$2\sin\left(x + \frac{\pi}{4}\right)\cdot\cos\left(2x - \frac{\pi}{4}\right) - 4\cos 2x \sin\frac{\pi}{4}\cos\left(2x - \frac{\pi}{4}\right) = 0 \Leftrightarrow$$
$$\cos\left(2x - \frac{\pi}{4}\right)\left[\sin\left(x + \frac{\pi}{4}\right) - \sqrt{2}\cos 2x\right] = 0.$$

1) $\cos\left(2x - \frac{\pi}{4}\right) = 0$; $2x - \frac{\pi}{4} = \frac{\pi}{2} + \pi k$, $k \in Z$; $x_1 = \frac{3\pi}{8} + \frac{\pi k}{2}$, $k \in Z$.

2) $\sin\left(x + \frac{\pi}{4}\right) - \sqrt{2}\cos 2x = 0 \Leftrightarrow \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} - \sqrt{2}\cos 2x = 0 \Leftrightarrow$

$$\Leftrightarrow \frac{\sqrt{2}}{2}(\sin x + \cos x) - \sqrt{2}\cos 2x = 0 \Leftrightarrow \sin x + \cos x - 2\cos 2x = 0 \Leftrightarrow$$
$$\Leftrightarrow \sin x + \cos x - 2\cos^2 x + 2\sin^2 x = 0 \Leftrightarrow.$$
$$\Leftrightarrow \sin x + \cos x + 2(\sin x + \cos x)(\sin x - \cos x) = 0 \Leftrightarrow$$
$$\Leftrightarrow (\sin x + \cos x)(1 + 2\sin x - 2\cos x) = 0.$$

2.1. $\sin x + \cos x = 0 \Leftrightarrow \mathrm{tg}x = -1 \Rightarrow x = -\frac{\pi}{4} + \pi n$, $n \in Z$.

2.2. $1 + 2\sin x - 2\cos x = 0$.

Переходим к половинному аргументу $\sin x = \frac{2\mathrm{tg}\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}}$, $\cos x = \frac{1 - \mathrm{tg}^2\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}}$ и делаем замену $t = \mathrm{tg}\frac{x}{2}$: $1 + \frac{4t}{1+t^2} - \frac{2-2t^2}{1+t^2} = 0 \Leftrightarrow 1 + t^2 + 4t - 2 + 2t^2 = 0 \Leftrightarrow 3t^2 + 4t - 1 = 0$;

$D = 4^2 + 4\cdot 3\cdot 1 = 28$; $\sqrt{D} = 2\sqrt{7}$; $t_1 = \frac{-4 - 2\sqrt{7}}{6} = -\frac{2+\sqrt{7}}{3}$; $t_2 = \frac{\sqrt{7}-2}{3}$.

$$\frac{x_1}{2} = -\mathrm{arctg}\frac{2+\sqrt{7}}{3} + \pi m \Rightarrow x_1 = -2\mathrm{arctg}\frac{2+\sqrt{7}}{3} + 2\pi m,\ m \in Z;$$
$$\frac{x_2}{2} = \mathrm{arctg}\frac{\sqrt{7}-2}{3} + \pi m \Rightarrow x_2 = 2\mathrm{arctg}\frac{\sqrt{7}-2}{3} + 2\pi m,\ m \in Z;$$