*Дана функция* y(x) = x3 - 3x2 + 1.

1) Область определения функции. Так как функция не имеет дроби или корня, то нет ограничения в области её определения.

D(y) = (−∞; +∞).

2) Четность и нечетность функции:

Проверим функцию - четна или нечетна с помощью соотношений f(x)=f(-x) и f(x)=-f(x). Итак, проверяем:

 $f\left(-x\right)=\left(-x\right)^{3}-3\*\left(-x\right)^{2}+1=-x^{3}-3x^{2}+1\ne f\left(x\right)\ne -f\left(x\right).$

3начит, функция не является ни чётной, ни нечётной.

3) Определим точки пересечения графика функции с осями координат.

Найдем точки пересечения с осью ординат Oy, для чего приравниваем x = 0: у = 03 – 3\*02 + 1 = 1.

Таким образом, точка пересечения с осью Oy имеет координаты (0;1).

Найдем точки пересечения с осью абсцисс Ox, для чего надо решить кубическое уравнение x3 - 3x2 + 1 = 0.

Для вычисления корней этого кубического уравнения используем тригонометрическую формулу Виета, которая работает для уравнений вида:

$$x^{3}+ax^{2}+bx+c=0.$$

Если уравнение не такого вида, то его можно получить поделив всё уравнение на коэффициент возле  $x^{3}.$

|  |  |
| --- | --- |
|  |  |

В нашем случае

|  |  |  |
| --- | --- | --- |
| a | = | −3, |
| b | = | 0 |

 и

|  |  |  |
| --- | --- | --- |
| c | = | 1. |

Теперь использовав формулы:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Q | = |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|

|  |  |  |  |
| --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| a | 2 |

 | −3b |

 |
| 9 |

 |  |

 и

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| R | = |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 2 |

|  |  |
| --- | --- |
| a | 3 |

 | −9ab | + | 27c |

 |
| 54 |

 |  |

 вычислим, что

|  |  |  |
| --- | --- | --- |
| Q | = | 1 |

и

|  |  |  |
| --- | --- | --- |
| R | = | −0,5. |

Далее по формуле

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| S | = |

|  |  |
| --- | --- |
| Q | 3 |

 | − |

|  |  |
| --- | --- |
| R | 2 |

 |  |

 видим, что

|  |  |  |
| --- | --- | --- |
| S | > | 0, поэтому уравнение будет иметь три вещественных корня. |

Которые вычисляются по следующим формулам:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| x | 1 |

 | = | −2 |

|  |  |
| --- | --- |
| √ | Q |

 | cos |

|  |  |  |
| --- | --- | --- |
| ( | ψ | ) |

 | − |

|  |
| --- |
| a |
| 3 |

 |

 |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| x | 2 |

 | = | −2 |

|  |  |
| --- | --- |
| √ | Q |

 | cos |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ( | ψ | + |

|  |
| --- |
| 2 |
| 3 |

 | π | ) |

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|  |
| --- |
| a |
| 3 |

 |

 |
|

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| x | 3 |

 | = | −2 |

|  |  |
| --- | --- |
| √ | Q |

 | cos |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ( | ψ | − |

|  |
| --- |
| 2 |
| 3 |

 | π | ) |

 | − |

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где

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  ψ | = |

|  |
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| 1 |
| 3 |

 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (arccos |

|  |
| --- |
| R |
|

|  |  |  |  |
| --- | --- | --- | --- |
| √ |

|  |  |
| --- | --- |
| Q | 3 |

 |

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 | ) . |

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Подставив наши числа в эти формулы, мы получим:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| x | 1 |

 | = | −0,5321 | ; |

|  |  |
| --- | --- |
| x | 2 |

 | = | 2,8794 | ; |

|  |  |
| --- | --- |
| x | 3 |

 | = | 0,6527. |

Отсюда имеем 3 точки пересечения с осью Ох:

(−0,5321; 0), (0,6527; 0) и (2,8794; 0).

4) Стационарные точки , интервалы возрастания и убывания функции , экстремумы функции

Исследуем функцию на экстремумы и монотонность. Для этого найдем первую производную функции: y’ = (x3 - 3x2 + 1)’ = 3x2 - 6х = 3х(x - 2).

Приравняем первую производную к нулю и найдем стационарные точки (в которых y′=0): 3х(x – 2) = 0, x1 = 0, х2 = 2.

Получили две критических точки:  х = 0 и х = 2.

Разобьем всю область определения функции на интервалы данными точками и определим знаки производной в каждом промежутке:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x = | -1 | 0 | 1 | 2 | 3 |
| y' = | 9 | 0 | -3 | 0 | 9 |

При x ∈ (0; 2) производная y′ < 0, поэтому функция убывает на данном промежутке.

При x ∈ (-∞; 0) U (2; ∞) производная y′ > 0, функция возрастает на данных промежутках. При этом x = 0 - точка локального максимума (функция возрастает, а потом убывает, x = 2 - точка локального минимума (функция убывает, а потом возрастает.

Значение функции в этих точках: у(0) = 1, у(2) = -3.

5) Дополнительные точки для построения графика функции y(x) = x3 - 3x2 + 1:

|  |  |
| --- | --- |
| **x** | **y** |
| **-2.0** | **-19** |
| **-1.5** | **-9.1** |
| **-1.0** | **-3** |
| **-0.5** | **0.1** |
| **0** | **1** |
| **0.5** | **0.4** |
| **1.0** | **-1** |
| **1.5** | **-2.4** |
| **2.0** | **-3** |
| **2.5** | **-2.1** |
| **3.0** | **1** |
| **3.5** | **7.1** |
| **4.0** | **17** |

6) По полученным данным строим график, и отметим характерные точки (пересечения с осями и экстремумы).

