Заданы координаты вершин треугольника А (-1;-2;4), В (-4;-1;2) и С (-5;6;-4). Найдите длину высоты ВD.

Находим уравнение стороны АC по точкам А (-1;-2;4) и С (-5;6;-4).

Вектор АC = (-5-(-1); 6-(-2); -4-4) = (-4; 8; -8).

Уравнение прямой АС (пусть она будет *L*0):

(х + 1)/(-4) = (у + 2)/8 = (z – 4)/(-8).

Находим расстояние от точки *B*(−4, −1, 2) до прямой *L*0:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| *L*0:    |

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| --- | --- | --- | --- |
| *x* | + |

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 | (1) |

**Решение.**

Чтобы найти расстояние от точки *B* до прямой *L*0 нужно:

* найти плоскость *α*, проходящей через точку *B* перпендикулярной прямой *L*0
* найти точку *M*1, которая является пересечением плоскости *α* с прямой *L*0.
* Найти расстояние между точками *B* и *M*1.

Уравнение плоскости *α*, проходящей через точку *B*(*x*0, *y*0, *z*0) и имеющий нормальный вектор *n*={*A*, *B*, *C*} представляется формулой:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| *A* | · |

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| --- | --- | --- | --- | --- | --- |
|  |

|  |  |  |
| --- | --- | --- |
| *x* | − | *x*0 |

 |  |

 | + | *B* | · |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |

|  |  |  |
| --- | --- | --- |
| *y* | − | *y*0 |

 |  |

 | + | *C* | · |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
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|  |  |  |
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| *z* | − | *z*0 |

 |  |

 | =0 |

 | (2) |

Направляющий вектор прямой *L*0 имеет следующие координаты:

|  |  |
| --- | --- |
| *q*0={*m*0, *p*0, *l*0}={−4, 8, −8} | (3) |

Для того, чтобы плоскость (2) была перпендикулярна прямой (1), нормальный вектор *n*={*A*, *B*, *C*} плоскости (2) должен быть коллинеарным направляющему вектору (3) прямой (1). Поэтому в качестве нормального вектора плоскости (2) можно взять вектор {*m*0, *p*0, *l*0}={−4, 8, −8}. Подставим координаты вектора *q*0 и координаты точки *B* в (2):

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| −4 |

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| 8 |

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 | (*y* | − |

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| ( |

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| −1 |

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| −8 |

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 | (*z* | − |

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 | ) | =0 |

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После упрощения получим уравнение плоскости, проходящей через точку *B* и перпендикулярной прямой *L*0:

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 |  *x* | + |

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| 8 |

 |

 |  *y* |

|  |  |
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| −8 |

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 |  *z* | + |

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 | (4) |

Для нахождения точки пересечения прямой (1) и плоскости (4) проще всего пользоваться параметрическим уравнением прямой (1).

Составим параметрическое уравнение прямой:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| *x* | + |

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| *y* | + |

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Выразим переменные *x*, *y*, *z* через параметр *t* :

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| *x*= |

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| 4 |

 | *t* |  , |
| *y*= |

|  |
| --- |
| −2 |

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 | *t* |  , |
| *z*= |

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| 8 |

 | *t* |  . |

 |  |

 | (5) |

Подставим значения *x, y, z* из выражения (5) в (4) и решим относительно *t*.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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Подставляя значение *t* в выражения (5), получим координаты точки *M*1:

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|

|  |
| --- |
| M1(−2,0,2). |

 |  |

Вычислим расстояние между точками *B* и *M*1

|  |  |
| --- | --- |
| |BM1|=√(−4−(−2))²+(−1−0)²+(2−2)² |  |
| |BM1|=√(−2)²+(−1)²+(0)²) =√5 |  |

**Ответ:** Расстояние от точки *B* до прямой (1):

 |BM1| =√5 ≈ 2,23606797.

|  |
| --- |
|  |