Заданы координаты вершин треугольника А (-1;-2;4), В (-4;-1;2) и С (-5;6;-4). Найдите длину высоты ВD.

Находим уравнение стороны АC по точкам А (-1;-2;4) и С (-5;6;-4).

Вектор АC = (-5-(-1); 6-(-2); -4-4) = (-4; 8; -8).

Уравнение прямой АС (пусть она будет *L*0):

(х + 1)/(-4) = (у + 2)/8 = (z – 4)/(-8).

Находим расстояние от точки *B*(−4, −1, 2) до прямой *L*0:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *L*0: | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | *x* | + | |  | | --- | | 1 | | | |  | | |  |  | | --- | --- | | |  | | --- | | −4 | | | | = | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | *y* | + | |  | | --- | | 2 | | | |  | | |  |  | | --- | --- | | |  | | --- | | 8 | | | | = | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | *z* | − | |  | | --- | | 4 | | | |  | | |  |  | | --- | --- | | |  | | --- | | −8 | | | | . | | (1) |

**Решение.**

Чтобы найти расстояние от точки *B* до прямой *L*0 нужно:

* найти плоскость *α*, проходящей через точку *B* перпендикулярной прямой *L*0
* найти точку *M*1, которая является пересечением плоскости *α* с прямой *L*0.
* Найти расстояние между точками *B* и *M*1.

Уравнение плоскости *α*, проходящей через точку *B*(*x*0, *y*0, *z*0) и имеющий нормальный вектор *n*={*A*, *B*, *C*} представляется формулой:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *A* | · | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | |  |  |  | | --- | --- | --- | | *x* | − | *x*0 | |  | | + | *B* | · | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | |  |  |  | | --- | --- | --- | | *y* | − | *y*0 | |  | | + | *C* | · | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | |  |  |  | | --- | --- | --- | | *z* | − | *z*0 | |  | | =0 | | (2) |

Направляющий вектор прямой *L*0 имеет следующие координаты:

|  |  |
| --- | --- |
| *q*0={*m*0, *p*0, *l*0}={−4, 8, −8} | (3) |

Для того, чтобы плоскость (2) была перпендикулярна прямой (1), нормальный вектор *n*={*A*, *B*, *C*} плоскости (2) должен быть коллинеарным направляющему вектору (3) прямой (1). Поэтому в качестве нормального вектора плоскости (2) можно взять вектор {*m*0, *p*0, *l*0}={−4, 8, −8}. Подставим координаты вектора *q*0 и координаты точки *B* в (2):

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | |  | | --- | | −4 | | | (*x* | − | |  |  |  |  | | --- | --- | --- | --- | | ( | |  | | --- | | −4 | | ) | | ) | + | |  |  | | --- | --- | | |  | | --- | | 8 | | | (*y* | − | |  |  |  |  | | --- | --- | --- | --- | | ( | |  | | --- | | −1 | | ) | | ) | |  |  | | --- | --- | | |  | | --- | | −8 | | | (*z* | − | |  |  | | --- | --- | | |  | | --- | | 2 | | | ) | =0 | |  |

После упрощения получим уравнение плоскости, проходящей через точку *B* и перпендикулярной прямой *L*0:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | |  | | --- | | −4 | | | *x* | + | |  |  | | --- | --- | | |  | | --- | | 8 | | | *y* | |  |  | | --- | --- | | |  | | --- | | −8 | | | *z* | + | |  |  | | --- | --- | | |  | | --- | | 8 | | | =0. | | (4) |

Для нахождения точки пересечения прямой (1) и плоскости (4) проще всего пользоваться параметрическим уравнением прямой (1).

Составим параметрическое уравнение прямой:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *t*= | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | *x* | + | |  | | --- | | 1 | | | |  | | |  |  | | --- | --- | | |  | | --- | | −4 | | | | , | | |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *t*= | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | *y* | + | |  | | --- | | 2 | | | |  | | |  |  | | --- | --- | | |  | | --- | | 8 | | | | , | | |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *t*= | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  |  |  |  | | --- | --- | --- | --- | | *z* | − | |  | | --- | | 4 | | | |  | | |  |  | | --- | --- | | |  | | --- | | −8 | | | | . | | |  |

Выразим переменные *x*, *y*, *z* через параметр *t* :

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | *x*= | |  | | --- | | −1 | | − | |  | | --- | | 4 | | *t* | , | | *y*= | |  | | --- | | −2 | | + | |  | | --- | | 8 | | *t* | , | | *z*= | |  | | --- | | 4 | | − | |  | | --- | | 8 | | *t* | . | |  | | (5) |

Подставим значения *x, y, z* из выражения (5) в (4) и решим относительно *t*.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | |  | | --- | | −4 | | | ( | |  |  | | --- | --- | | |  | | --- | | −4 | | | *t* | |  |  | | --- | --- | | |  | | --- | | −1 | | | ) | + | |  |  | | --- | --- | | |  | | --- | | 8 | | | ( | |  |  | | --- | --- | | |  | | --- | | 8 | | | *t* | |  |  | | --- | --- | | |  | | --- | | −2 | | | ) | |  |  | | --- | --- | | |  | | --- | | −8 | | | ( | |  |  | | --- | --- | | |  | | --- | | −8 | | | *t* | + | |  |  | | --- | --- | | |  | | --- | | 4 | | | ) | + | |  |  | | --- | --- | | |  | | --- | | 8 | | | = | |  |  | | --- | --- | | |  | | --- | | 0 | | | |  |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | 16 | |  | | --- | |  | |  | |  | | | | *t* | + | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | 64 | |  | | --- | |  | |  | |  | | | | *t* | + | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | 64 | |  | | --- | |  | |  | |  | | | | *t* | + | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | 4 | |  | | --- | |  | |  | |  | | | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | −16 | |  | | --- | |  | |  | |  | | | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | −32 | |  | | --- | |  | |  | |  | | | | + | |  |  | | --- | --- | | |  | | --- | | 8 | | | = | |  |  | | --- | --- | | |  | | --- | | 0 | | | |  |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | *t*= | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 1 | |  | | 4 | | | | |  |

Подставляя значение *t* в выражения (5), получим координаты точки *M*1:

|  |  |  |
| --- | --- | --- |
| |  | | --- | | M1(−2,0,2). | |  |

Вычислим расстояние между точками *B* и *M*1

|  |  |
| --- | --- |
| |BM1|=√(−4−(−2))²+(−1−0)²+(2−2)² |  |
| |BM1|=√(−2)²+(−1)²+(0)²) =√5 |  |

**Ответ:** Расстояние от точки *B* до прямой (1):

|BM1| =√5 ≈ 2,23606797.

|  |
| --- |
|  |