

$$Ax + By + C = 0; y = -\frac{Ax + C}{B}$$

$$\begin{aligned} R &= \sqrt{(x - x_0)^2 + (y - y_0)^2} = \sqrt{(x - x_0)^2 + \left(-\frac{Ax + C}{B} - y_0\right)^2} = \\ &= \sqrt{x^2 - 2x * x_0 + x_0^2 + \left(\frac{A}{B}\right)^2 x^2 + 2\left(\frac{A}{B} * \frac{C + By_0}{B}\right) * x + \left(\frac{C + By_0}{B}\right)^2} = R(x) \\ \frac{dR(x)}{dx} &= \frac{1}{2R} * \left(2x - 2x_0 + 2\left(\frac{A}{B}\right)^2 x + 2\left(\frac{A}{B} * \frac{C + By_0}{B}\right)\right) = 0 \end{aligned}$$

$$x\left(1 + \frac{A^2}{B^2}\right) = x_0 - \frac{A}{B} * \frac{C + By_0}{B}; x = \frac{x_0 - \frac{A}{B} * \frac{C + By_0}{B}}{1 + \frac{A^2}{B^2}} = \frac{x_0 B^2 - A(C + By_0)}{B^2 + A^2}$$

$$\begin{aligned} R &= \sqrt{(x - x_0)^2 + \left(-\frac{Ax + C}{B} - y_0\right)^2} = \sqrt{\left(\frac{x_0 B^2 - A(C + By_0)}{B^2 + A^2} - x_0\right)^2 + \left(-\frac{A\left(\frac{x_0 B^2 - A(C + By_0)}{B^2 + A^2}\right) + C}{B} - y_0\right)^2} = \\ &= \sqrt{\left(\frac{x_0 B^2 - A(C + By_0) - x_0 B^2 - x_0 A^2}{B^2 + A^2}\right)^2 + \left(\frac{x_0 AB^2 - A^2(C + By_0) + CB^2 + CA^2 + y_0 B^3 + y_0 BA^2}{(B^2 + A^2)B}\right)^2} = \\ &= \frac{\sqrt{(A(C + By_0) + x_0 A^2)^2 + (x_0 AB + CB + y_0 B^2)^2}}{B^2 + A^2} = \frac{\sqrt{(A(Ax_0 + By_0 + C))^2 + (B(Ax_0 + By_0 + C))^2}}{B^2 + A^2} = \\ &= \frac{\sqrt{(Ax_0 + By_0 + C)^2 * (B^2 + A^2)}}{B^2 + A^2} = \frac{|Ax_0 + By_0 + C|}{\sqrt{B^2 + A^2}} \end{aligned}$$

From iuv