

$$\frac{x}{x-2y} - \frac{4y^2}{x^2-2xy} = \frac{x}{x-2y} - \frac{4y^2}{x(x-2y)} = \frac{x^2}{(x-2y)x} - \frac{4y^2}{x(x-2y)} = \frac{x^2-4y^2}{(x-2y)x} = \frac{(x+2y)(x-2y)}{(x-2y)x} = \frac{x+2y}{x}$$

$$\frac{9x^2}{3xy-y^2} - \frac{y}{3x-y} = \frac{9x^2}{y(3x-y)} - \frac{y}{3x-y} = \frac{9x^2}{y(3x-y)} - \frac{y^2}{(3x-y)y} = \frac{9x^2-y^2}{y(3x-y)} = \frac{(3x+y)(3x-y)}{y(3x-y)} = \frac{3x+y}{y}$$

$$\begin{aligned} \frac{x}{x-4} - \frac{x+2}{x-2} &= \frac{x(x-2)}{(x-4)(x-2)} - \frac{(x+2)(x-4)}{(x-2)(x-4)} = \frac{x(x-2)-(x+2)(x-4)}{(x-4)(x-2)} = \frac{(x^2-2x)-(x^2-4x+2x-8)}{(x-4)(x-2)} = \\ &= \frac{(x^2-2x)-(x^2-2x-8)}{(x-4)(x-2)} = \frac{x^2-2x-x^2+2x+8}{(x-4)(x-2)} = \frac{8}{(x-4)(x-2)} \end{aligned}$$

$$\begin{aligned} 6y + \frac{12y}{6y-1} - 1 &= (6y-1) + \frac{12y}{6y-1} = \frac{(6y-1)^2}{6y-1} + \frac{12y}{6y-1} = \frac{(6y-1)^2+12y}{6y-1} = \frac{(36y^2-12y+1)+12y}{6y-1} = \\ &= \frac{36y^2-12y+1+12y}{6y-1} = \frac{36y^2+1}{6y-1} \end{aligned}$$

$$\begin{aligned} \frac{3}{x+3} + \frac{3}{x^2-3x} + \frac{2x}{9-x^2} &= \frac{3}{x+3} + \frac{3}{x^2-3x} + \frac{2x}{-x^2+9} = \frac{3}{x+3} + \frac{3}{x^2-3x} - \frac{2x}{x^2-9} = \frac{3}{x+3} + \frac{3}{x(x-3)} - \frac{2x}{(x-3)(x+3)} = \\ &= \frac{3x(x-3)}{(x+3)x(x-3)} + \frac{3(x+3)}{x(x-3)(x+3)} - \frac{2x^2}{(x-3)(x+3)x} = \frac{3x(x-3)+3(x+3)-2x^2}{(x+3)x(x-3)} = \\ &= \frac{(3x)(x-3)+3(x+3)-2x^2}{(x+3)x(x-3)} = \frac{(3x^2-9x)+(3x+9)-2x^2}{(x+3)x(x-3)} = \frac{3x^2-9x+3x+9-2x^2}{(x+3)x(x-3)} = \\ &= \frac{x^2-6x+9}{(x+3)x(x-3)} = \frac{(x-3)(x-3)}{(x+3)x(x-3)} = \frac{x-3}{(x+3)x} \end{aligned}$$