Дана матрица 

Обозначим через А — матрицу коэффициентов при неизвестных; X — матрицу-столбец неизвестных; B - матрицу-столбец свободных членов:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| 2 | -1 | 0 |
| 1 | -2 | -1 |
| 0 | 1 | 1 |

 |  |

 |  |

Вектор B:
BT=(-1,-2,-2)
С учетом этих обозначений данная система уравнений принимает следующую матричную форму: А\*Х = B.
Если матрица А — невырожденная (ее определитель отличен от нуля, то она имеет обратную матрицу А-1. Умножив обе части уравнения на А-1, получим: А-1\*А\*Х = А-1\*B, А-1\*А=Е.
Это равенство называется **матричной записью решения системы линейных уравнений**. Для нахождения решения системы уравнений необходимо вычислить обратную матрицу А-1.
Система будет иметь решение, если определитель матрицы A отличен от нуля.
Найдем главный определитель.
∆=2•(-2•1-1•(-1))-1•(-1•1-1•0)+0•(-1•(-1)-(-2•0))=-1
Итак, определитель -1 ≠ 0, поэтому продолжаем решение. Для этого найдем обратную матрицу через алгебраические дополнения.
Пусть имеем невырожденную матрицу А:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A= |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| a11 | a12 | a13 |
| a21 | a22 | a23 |
| a31 | a32 | a33 |

 |  |

 |  |

Тогда:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=A%5e%7b-1%7d%20=%20\frac%7b1%7d%7b\Delta%20%7d |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| A11 | A21 | A31 |
| A12 | A22 | A32 |
| A13 | A23 | A33 |

 |  |

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где Aij — алгебраическое дополнение элемента aij в определителе матрицы А, которое является произведением (—1)i+j на минор (определитель) *n-1* порядка, полученный вычеркиванием *i-й* строки и *j-го* столбца в определителе матрицы А.
**Транспонированная матрица** к матрице A имеет вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT= |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| 2 | 1 | 0 |
| -1 | -2 | 1 |
| 0 | -1 | 1 |

 |  |

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Вычисляем алгебраические дополнения.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT1,1=(-1)1+1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| -2 | 1 |
| -1 | 1 |

 |  |

 |  |

∆1,1=(-2•1-(-1•1))=-1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT1,2=(-1)1+2 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| -1 | 1 |
| 0 | 1 |

 |  |

 |  |

∆1,2=-(-1•1-0•1)=1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT1,3=(-1)1+3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| -1 | -2 |
| 0 | -1 |

 |  |

 |  |

∆1,3=(-1•(-1)-0•(-2))=1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT2,1=(-1)2+1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 1 | 0 |
| -1 | 1 |

 |  |

 |  |

∆2,1=-(1•1-(-1•0))=-1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT2,2=(-1)2+2 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 2 | 0 |
| 0 | 1 |

 |  |

 |  |

∆2,2=(2•1-0•0)=2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT2,3=(-1)2+3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 2 | 1 |
| 0 | -1 |

 |  |

 |  |

∆2,3=-(2•(-1)-0•1)=2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT3,1=(-1)3+1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 1 | 0 |
| -2 | 1 |

 |  |

 |  |

∆3,1=(1•1-(-2•0))=1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT3,2=(-1)3+2 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 2 | 0 |
| -1 | 1 |

 |  |

 |  |

∆3,2=-(2•1-(-1•0))=-2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT3,3=(-1)3+3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 2 | 1 |
| -1 | -2 |

 |  |

 |  |

∆3,3=(2•(-2)-(-1•1))=-3
Из полученных алгебраических дополнений составим присоединенную матрицу C:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C= |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| -1 | 1 | 1 |
| -1 | 2 | 2 |
| 1 | -2 | -3 |

 |  |

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**Вычислим обратную матрицу**:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=A%5e%7b-1%7d=\frac%7b1%7d%7b-1%7d |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| -1 | 1 | 1 |
| -1 | 2 | 2 |
| 1 | -2 | -3 |

 |  |

 |  |

**Вектор результатов X**
X=A-1 • B

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=X=\frac%7b1%7d%7b-1%7d |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |  |
| --- | --- | --- |
| -1 | 1 | 1 |
| -1 | 2 | 2 |
| 1 | -2 | -3 |

 |  |

 | \* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|

|  |
| --- |
| -1 |
| -2 |
| -2 |

 |  |

 |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=X=\frac%7b1%7d%7b-1%7d |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|

|  |
| --- |
| (-1(-1))+(1(-2))+(1(-2)) |
| (-1(-1))+(2(-2))+(2(-2)) |
| (1(-1))+(-2(-2))+(-3(-2)) |

 |  |

 |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=X=\frac%7b1%7d%7b-1%7d |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|

|  |
| --- |
| -3 |
| -7 |
| 9 |

 |  |

 |  |

XT=(3,7,-9)
x1=-3 / (-1)=3
x2=-7 / (-1)=7
x3=9 / (-1)=-9
**Проверка**.
2•3-1•7+0•(-9)=-1
1•3-2•7-1•(-9)=-2
0•3+1•7+1•(-9)=-2