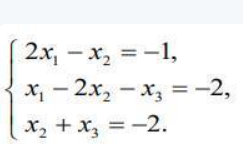
Дана матрица 

Обозначим через А — матрицу коэффициентов при неизвестных; X — матрицу-столбец неизвестных; B - матрицу-столбец свободных членов:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | 2 | -1 | 0 | | 1 | -2 | -1 | | 0 | 1 | 1 | |  | |  |

Вектор B:  
BT=(-1,-2,-2)  
С учетом этих обозначений данная система уравнений принимает следующую матричную форму: А\*Х = B.  
Если матрица А — невырожденная (ее определитель отличен от нуля, то она имеет обратную матрицу А-1. Умножив обе части уравнения на А-1, получим: А-1\*А\*Х = А-1\*B, А-1\*А=Е.  
Это равенство называется **матричной записью решения системы линейных уравнений**. Для нахождения решения системы уравнений необходимо вычислить обратную матрицу А-1.  
Система будет иметь решение, если определитель матрицы A отличен от нуля.  
Найдем главный определитель.  
∆=2•(-2•1-1•(-1))-1•(-1•1-1•0)+0•(-1•(-1)-(-2•0))=-1  
Итак, определитель -1 ≠ 0, поэтому продолжаем решение. Для этого найдем обратную матрицу через алгебраические дополнения.  
Пусть имеем невырожденную матрицу А:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A= | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | a11 | a12 | a13 | | a21 | a22 | a23 | | a31 | a32 | a33 | |  | |  |

Тогда:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=A%5e%7b-1%7d%20=%20\frac%7b1%7d%7b\Delta%20%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | A11 | A21 | A31 | | A12 | A22 | A32 | | A13 | A23 | A33 | |  | |  |

где Aij — алгебраическое дополнение элемента aij в определителе матрицы А, которое является произведением (—1)i+j на минор (определитель) *n-1* порядка, полученный вычеркиванием *i-й* строки и *j-го* столбца в определителе матрицы А.  
**Транспонированная матрица** к матрице A имеет вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT= | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | 2 | 1 | 0 | | -1 | -2 | 1 | | 0 | -1 | 1 | |  | |  |

Вычисляем алгебраические дополнения.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT1,1=(-1)1+1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -2 | 1 | | -1 | 1 | |  | |  |

∆1,1=(-2•1-(-1•1))=-1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT1,2=(-1)1+2 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | 1 | | 0 | 1 | |  | |  |

∆1,2=-(-1•1-0•1)=1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT1,3=(-1)1+3 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | -1 | -2 | | 0 | -1 | |  | |  |

∆1,3=(-1•(-1)-0•(-2))=1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT2,1=(-1)2+1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 1 | 0 | | -1 | 1 | |  | |  |

∆2,1=-(1•1-(-1•0))=-1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT2,2=(-1)2+2 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 0 | | 0 | 1 | |  | |  |

∆2,2=(2•1-0•0)=2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT2,3=(-1)2+3 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 1 | | 0 | -1 | |  | |  |

∆2,3=-(2•(-1)-0•1)=2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT3,1=(-1)3+1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 1 | 0 | | -2 | 1 | |  | |  |

∆3,1=(1•1-(-2•0))=1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT3,2=(-1)3+2 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 0 | | -1 | 1 | |  | |  |

∆3,2=-(2•1-(-1•0))=-2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AT3,3=(-1)3+3 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 1 | | -1 | -2 | |  | |  |

∆3,3=(2•(-2)-(-1•1))=-3  
Из полученных алгебраических дополнений составим присоединенную матрицу C:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C= | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -1 | 1 | 1 | | -1 | 2 | 2 | | 1 | -2 | -3 | |  | |  |

**Вычислим обратную матрицу**:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=A%5e%7b-1%7d=\frac%7b1%7d%7b-1%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -1 | 1 | 1 | | -1 | 2 | 2 | | 1 | -2 | -3 | |  | |  |

**Вектор результатов X**  
X=A-1 • B

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=X=\frac%7b1%7d%7b-1%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | -1 | 1 | 1 | | -1 | 2 | 2 | | 1 | -2 | -3 | |  | | \* | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  | | --- | | -1 | | -2 | | -2 | |  | |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=X=\frac%7b1%7d%7b-1%7d | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  | | --- | | (-1(-1))+(1(-2))+(1(-2)) | | (-1(-1))+(2(-2))+(2(-2)) | | (1(-1))+(-2(-2))+(-3(-2)) | |  | |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=X=\frac%7b1%7d%7b-1%7d | |  |  |  |  |  | | --- | --- | --- | --- | --- | | |  | | --- | | -3 | | -7 | | 9 | |  | |  |

XT=(3,7,-9)  
x1=-3 / (-1)=3  
x2=-7 / (-1)=7  
x3=9 / (-1)=-9  
**Проверка**.  
2•3-1•7+0•(-9)=-1  
1•3-2•7-1•(-9)=-2  
0•3+1•7+1•(-9)=-2