$$\log_{\sqrt{26}+5}\left(x^2+2x+2\sqrt{2}\right)=\log_{\sqrt{26}-5}\left(6-x^2-2x-2\sqrt{2}\right)$$

ОДЗ:

$$\begin{cases} x^2+2x+2\sqrt{2}>0 \\ 6-x^2-2x-2\sqrt{2}>0 \end{cases} \Leftrightarrow \begin{cases} \left(x+1\right)^2+2\sqrt{2}-1>0 \\ -\left(x+1\right)^2+7-2\sqrt{2}>0 \end{cases} \Leftrightarrow \begin{cases} x\in\mathbb{R} \\ \left(x+1\right)^2<7-2\sqrt{2} \end{cases}$$

$$-\sqrt{7-2\sqrt{2}}<x+1<\sqrt{7-2\sqrt{2}} \quad\Rightarrow\quad \boxed{-1-\sqrt{7-2\sqrt{2}}<x<\sqrt{7-2\sqrt{2}}-1}$$

Не трудно заметить, что $\sqrt{26}-5=\dfrac{1}{\sqrt{26}+5}$, тогда $\dfrac{1}{\sqrt{26}+5}=\left(\sqrt{26}+5\right)^{-1}$

$$\log_{\sqrt{26}+5}\left(x^2+2x+2\sqrt{2}\right)=\log_{\left(\sqrt{26}+5\right)^{-1}}\left(6-x^2-2x-2\sqrt{2}\right)$$
$$\log_{\sqrt{26}+5}\left(x^2+2x+2\sqrt{2}\right)=-\log_{\sqrt{26}+5}\left(6-x^2-2x-2\sqrt{2}\right)$$
$$\log_{\sqrt{26}+5}\left(x^2+2x+2\sqrt{2}\right)=\log_{\sqrt{26}+5}\left(\frac{1}{6-x^2-2x-2\sqrt{2}}\right)$$
$$x^2+2x+2\sqrt{2}=\frac{1}{6-x^2-2x-2\sqrt{2}}$$

Пусть $x^2+2x+2\sqrt{2}=t$.

$$t=\frac{1}{6-t}$$

$$t\left(6-t\right)=1$$
$$t^2-6t+1=0$$
$$\left(t-3\right)^2-8=0$$
$$\left(t-3\right)^2=8$$
$$t-3=\pm2\sqrt{2}$$
$$t=3\pm2\sqrt{2}$$

Вернёмся к обратной замене

$$x^2 + 2x + 2\sqrt{2} = 3 + 2\sqrt{2}$$
$$x^2 + 2x - 3 = 0$$
$$x_1 = -3;$$
$$x_2 = 1.$$

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$$x^2 + 2x + 2\sqrt{2} = 3 - 2\sqrt{2}$$

$$x^2 + 2x + 3 + 4\sqrt{2} = 0$$
$$D = 4 - 4\left(3 + 4\sqrt{2}\right) = 4\left(-2 - 4\sqrt{2}\right) < 0$$

Ответ: $-3; \ 1.$