$\sin^3 x + \cos^3 x = 1 + \sin 2x$

$(\sin x + \cos x)(\sin^2 x - \sin x \cdot \cos x + \cos^2 x) = (\sin x + \cos x)^2$

$(\sin x + \cos x)(\sin^2 x - \sin x \cdot \cos x + \cos^2 x - \sin x - \cos x) = 0.$

$\operatorname{tg} x = -1$

$x_1 = -\frac{\pi}{4} + \pi k, k \in Z.$

$\overset{1}{\sin^2 x} - \overset{2}{\sin x \cdot \cos x} + \overset{3}{\cos^2 x} - \overset{4}{\sin x} - \cos x = 0.$

$\sin^2 x + \cos^2 x + 2\sin x \cdot \cos x - 3\sin x \cdot \cos x - (\sin x + \cos x) = 0.$

$(\sin x + \cos x)^2 - (\sin x + \cos x) - 3\sin x \cdot \cos x = 0.$

Пусть $\sin x + \cos x = t$ $(|t| \le \sqrt{2})$, тогда имеем:

$(\sin x + \cos x)^2 = t^2; \Rightarrow 1 + 2\sin x \cdot \cos x = t^2 \Rightarrow \sin x \cdot \cos x = \frac{t^2 - 1}{2}.$

$t^2 - t - 3 \cdot \frac{t^2 - 1}{2} = 0 \mid \cdot 2$

$2t^2 - 2t - 3t^2 + 3 = 0.$

$-t^2 - 2t + 3 = 0$

$t^2 + 2t - 3 = 0$

$D = 4 + 12 = 16$

$t_1 = \frac{-2+4}{2} = 1$

$t_2 = \frac{-2-4}{2} = -3 - \notin [-\sqrt{2}; \sqrt{2}].$

Замена:

$\sin x + \cos x = 1$

$\sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = 1$

$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x + \frac{\pi}{4} = (-1)^n \cdot \frac{\pi}{4} + \pi n, n \in Z$

$x = (-1)^n \cdot \frac{\pi}{4} - \frac{\pi}{4} + \pi n, n \in Z.$