

$$\sqrt[3]{10+6\sqrt{3}-\sqrt{3}} = (1+\sqrt{3})-\sqrt{3} = 1+\sqrt{3}-\sqrt{3} = 1$$

$$\begin{aligned} (\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}})^6 &= \left(\sqrt{(\sqrt{1,5}-\sqrt{0,5})^2} + \sqrt{(\sqrt{1,5}+\sqrt{0,5})^2} \right)^6 = \\ &= \left((\sqrt{1,5}-\sqrt{0,5}) + (\sqrt{1,5}+\sqrt{0,5}) \right)^6 = (\sqrt{1,5}-\sqrt{0,5}+\sqrt{1,5}+\sqrt{0,5})^6 = \\ &= (\sqrt{1,5}+\sqrt{1,5})^6 = (2\sqrt{1,5})^6 = \left(2\sqrt{\frac{15}{10}} \right)^6 = \left(\frac{2\sqrt{6}}{2} \right)^6 = \sqrt{6}^6 = \sqrt{6^6} = \\ &= 6^3 = 216 \end{aligned}$$

$$\begin{aligned} \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+2} &= \frac{1-\sqrt{2}}{1-\sqrt{2}^2} + \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} + \frac{\sqrt{3}-2}{\sqrt{3}^2-2^2} = \\ &= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-2}{3-2^2} = -\frac{1-\sqrt{2}}{1} - \frac{\sqrt{2}-\sqrt{3}}{1} + \frac{\sqrt{3}-2}{3-2^2} = \\ &= -(1-\sqrt{2}) - (\sqrt{2}-\sqrt{3}) + \frac{\sqrt{3}-2}{3-2^2} = -1+\sqrt{2}-\sqrt{2}+\sqrt{3} + \frac{\sqrt{3}-2}{3-2^2} = -1+\sqrt{3} + \frac{\sqrt{3}-2}{3-2^2} = \\ &= -1+\sqrt{3} + \frac{\sqrt{3}-2}{-1} = -1+\sqrt{3} - \frac{\sqrt{3}-2}{1} = -1+\sqrt{3}-\sqrt{3}+2 = -1+2 = 1 \end{aligned}$$