

$$\left(\begin{array}{l} L = -\frac{d^2}{dx^2} + 1 \\ Lf = \lambda f \\ -\frac{d^2 f}{dx^2} + f = \lambda f \\ -k^2 + 1 - \lambda = 0 \end{array} \right) \Rightarrow \left\{ \begin{array}{l} \lambda < 1; \\ f = A * e^{x\sqrt{1-\lambda}} + B * e^{-x\sqrt{1-\lambda}} \\ \lambda > 1; \\ f = A * \sin(\sqrt{\lambda-1} * x) + B * \cos(\sqrt{\lambda-1} * x) \\ \lambda = 1; \\ f = A * x + B \end{array} \right.$$

I; $\lambda < 1$

$$f = A * e^{x\sqrt{1-\lambda}} + B * e^{-x\sqrt{1-\lambda}}$$

$$f' = A * \sqrt{1-\lambda} * e^{x\sqrt{1-\lambda}} - B * \sqrt{1-\lambda} * e^{-x\sqrt{1-\lambda}}$$

$$f(-\pi) = A * e^{-\pi\sqrt{1-\lambda}} + B * e^{\pi\sqrt{1-\lambda}}$$

$$f(\pi) = A * e^{\pi\sqrt{1-\lambda}} + B * e^{-\pi\sqrt{1-\lambda}}$$

$$f'(\pi) = A * \sqrt{1-\lambda} * e^{\pi\sqrt{1-\lambda}} - B * \sqrt{1-\lambda} * e^{-\pi\sqrt{1-\lambda}}$$

$$\begin{cases} f(-\pi) = 0 \\ f'(\pi) + kf(\pi) = 0 \end{cases} \Rightarrow \begin{cases} A * e^{-\pi\sqrt{1-\lambda}} + B * e^{\pi\sqrt{1-\lambda}} = 0 \\ A * \sqrt{1-\lambda} * e^{\pi\sqrt{1-\lambda}} - B * \sqrt{1-\lambda} * e^{-\pi\sqrt{1-\lambda}} + kA * e^{\pi\sqrt{1-\lambda}} + kB * e^{-\pi\sqrt{1-\lambda}} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} A = -B * e^{2\pi\sqrt{1-\lambda}} \\ A * (k + \sqrt{1-\lambda}) * e^{\pi\sqrt{1-\lambda}} + B * (k - \sqrt{1-\lambda}) * e^{-\pi\sqrt{1-\lambda}} = 0 \end{cases} \Rightarrow \begin{cases} A = -B * e^{2\pi\sqrt{1-\lambda}} \\ e^{2\pi\sqrt{1-\lambda}} * (k + \sqrt{1-\lambda}) = (k - \sqrt{1-\lambda}) * e^{-2\pi\sqrt{1-\lambda}} \end{cases} \Rightarrow \begin{cases} k > 0 \\ \sqrt{1-\lambda} = 0 \Rightarrow \{\emptyset\} \\ \lambda < 1!!! \end{cases}$$

II; $\lambda > 1$

$$f = A * \sin(x\sqrt{\lambda-1}) + B * \cos(x\sqrt{\lambda-1})$$

$$f' = A * \sqrt{\lambda-1} * \cos(x\sqrt{\lambda-1}) - B * \sqrt{\lambda-1} * \sin(x\sqrt{\lambda-1})$$

$$f(-\pi) = A * \sin(-\pi\sqrt{\lambda-1}) + B * \cos(-\pi\sqrt{\lambda-1}) = -A * \sin(\pi\sqrt{\lambda-1}) + B * \cos(\pi\sqrt{\lambda-1})$$

$$f(\pi) = A * \sin(\pi\sqrt{\lambda-1}) + B * \cos(\pi\sqrt{\lambda-1})$$

$$f'(\pi) = A * \sqrt{\lambda-1} * \cos(\pi\sqrt{\lambda-1}) - B * \sqrt{\lambda-1} * \sin(\pi\sqrt{\lambda-1})$$

$$\begin{cases} f(-\pi) = 0 \\ f'(\pi) + kf(\pi) = 0 \end{cases} \Rightarrow \begin{cases} -A * \sin(\pi\sqrt{\lambda-1}) + B * \cos(\pi\sqrt{\lambda-1}) = 0 \\ A * \sqrt{\lambda-1} * \cos(\pi\sqrt{\lambda-1}) - B * \sqrt{\lambda-1} * \sin(\pi\sqrt{\lambda-1}) + kA * \sin(\pi\sqrt{\lambda-1}) + kB * \cos(\pi\sqrt{\lambda-1}) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \pi\sqrt{\lambda-1} = \alpha \\ A = B * \frac{\cos(\alpha)}{\sin(\alpha)} \\ \sqrt{\lambda-1} * \cos^2(\alpha) - \sqrt{\lambda-1} * \sin^2(\alpha) + 2k \cos(\alpha) \sin(\alpha) = 0 \end{cases} \Rightarrow \begin{cases} \pi\sqrt{\lambda-1} = \alpha \\ A = \frac{B}{\operatorname{tg}(\alpha)} \\ \sqrt{\lambda-1} * \cos(2\alpha) + k \sin(2\alpha) = 0 \end{cases} \Rightarrow \begin{cases} \pi\sqrt{\lambda-1} = \alpha \\ A = \frac{B}{\operatorname{tg}(\alpha)} \\ \operatorname{tg}(2\pi\sqrt{\lambda-1}) = -\frac{\sqrt{\lambda-1}}{k} \end{cases}$$

$$\begin{cases} A^2 + B^2 = 1 \\ A = B * \frac{\cos(\alpha)}{\sin(\alpha)} \end{cases} \Rightarrow \begin{cases} A = \cos(\alpha) \\ B = \sin(\alpha) \end{cases}$$

$$f = A * \sin(x\sqrt{\lambda-1}) + B * \cos(x\sqrt{\lambda-1}) = \cos(\alpha) * \sin(x\sqrt{\lambda-1}) + \sin(\alpha) * \cos(x\sqrt{\lambda-1}) = \sin(x\sqrt{\lambda-1} + \alpha) = \sin(x\sqrt{\lambda-1} + \pi\sqrt{\lambda-1})$$

$$\begin{bmatrix} f = \sin((x + \pi)\sqrt{\lambda-1}) \\ \operatorname{tg}(2\pi\sqrt{\lambda-1}) = -\frac{\sqrt{\lambda-1}}{k} \end{bmatrix}$$

