

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \operatorname{ctg}^2 2x \, dx$$

$$\int \operatorname{ctg}^2 2x \, dx = \int \frac{\operatorname{ctg} t^2}{2} dt =$$

$$= \frac{1}{2} \int \operatorname{ctg} t^2 \, dt = \frac{1}{2} \int \csc t^2 - 1 \, dt =$$

$$= \frac{1}{2} \left(\int \csc t^2 \, dt - \int 1 \, dt \right) =$$

$$= \frac{1}{2} (-\operatorname{ctg}(t) - t) \stackrel{t=2x}{=} \frac{1}{2} (-\operatorname{ctg} 2x - 2x) =$$

$$= -\frac{\operatorname{ctg} 2x}{2} - x$$

$$\left(-\frac{\operatorname{ctg} 2x}{2} - x \right) \bigg|_{\frac{\pi}{8}}^{\frac{\pi}{4}} = -\frac{\operatorname{ctg} 2 \frac{\pi}{4}}{2} - \frac{\pi}{4} -$$

$$\left(-\frac{\operatorname{ctg} 2 \frac{\pi}{8}}{2} - \frac{\pi}{8} \right) = -\frac{0}{2} - \frac{\pi}{4} - \left(-\frac{1}{2} - \frac{\pi}{8} \right) =$$

$$= \frac{1}{2} - \frac{\pi}{8}$$